

Instructions: Try these problems from the textbook as practice (answers below).

Problems

9-36. Compute the first-order partial derivatives of each function below.

9. $f(x, y) = x^4 + 5xy^3$

11. $f(x, y) = x^3 \sin(y)$

13. $z = \ln(x + t^2)$

15. $f(x, y) = ye^{xy}$

17. $g(x, y) = x(x + x^2y)^5$

19. $f(x, y) = \frac{ax + by}{cx + dy}$

21. $g(u, v) = (u^2v - v^3)^5$

23. $R(p, q) = \tan^{-1}(pq^2)$

25. $F(x, y) = \int_y^x \cos(e^t) dt$

27. $f(x, y, z) = xy^2e^{-xz}$

29. $w = \ln(x + 2y + 3z)$

31. $p = \sqrt{t^4 + u^2} \cos(v)$

33. $h(x, y, z, t) = x^2y \cos(z/t)$

35. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

37-40. Compute the indicated value.

37. $R(s, t) = te^{s/t}$; $R_t(0, 1)$

39. $f(x, y, z) = \ln \left(\frac{1 - \sqrt{x^2 + y^2 + z^2}}{1 + \sqrt{x^2 + y^2 + z^2}} \right)$; $f_y(1, 2, 2)$

41-44. Use the Implicit Function Theorem to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

41. $x^2 + 2y^2 + 3z^2 = 1$

43. $e^z = xyz$

47. Compute all second-order partial derivatives of $f(x, y) = x^4y - x^3y$.

51. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(a) $z = f(x) + g(y)$

(b) $z = f(x + y)$

57-64. Compute the indicated partial derivative(s).

57. $f(x, y) = x^4y^2 - x^3y$; f_{xxx}, f_{xyx}

59. $f(x, y, z) = e^{xyz^2}$; f_{xyz}

61. $W = \sqrt{u + v^2}$; $\frac{\partial^3 W}{\partial u^2 \partial v}$

Answers

9. $f_x = 4x^3 + 5y^3$, $f_y = 15xy^2$
11. $f_x = 3x^2 \sin(y)$, $f_y = x^3 \cos(y)$
13. $z_x = \frac{1}{x+t^2}$, $z_t = \frac{2t}{x+t^2}$
15. $f_x = y^2 e^{xy}$, $f_y = (1+xy)e^{xy}$
17. $g_x = x^5(1+xy)^4(x+10xy+x^2y+5)$, $g_y = 5x^3(x+x^2y)^4$
19. $f_x = \frac{(ad-bc)y}{(cx+dy)^2}$, $f_y = \frac{(bc-ad)x}{(cx+dy)^2}$
21. $g_u = 10uv(u^2v-v^3)^4$, $g_v = 5(u^2-3v^2)(u^2v-v^3)^4$
23. $R_p = \frac{q^2}{1+p^2q^4}$, $R_q = \frac{2pq}{1+p^2q^4}$
25. $F_x = \cos(e^x)$, $F_y = -\cos(e^y)$
27. $f_x = (1-xz)y^2e^{-xz}$, $f_y = 2xye^{-xz}$, $f_z = -x^2y^2e^{-xz}$
29. $w_x = \frac{1}{x+2y+3x}$, $w_y = \frac{2}{x+2y+3x}$, $w_z = \frac{3}{x+2y+3x}$
31. $p_t = 2t^3(t^4+u^2\cos(v))^{-\frac{1}{2}}$, $p_u = u\cos(v)(t^4+u^2\cos(v))^{-\frac{1}{2}}$, $p_v = -\frac{1}{2}u^2\sin(v)(t^4+u^2\cos(v))^{-\frac{1}{2}}$
33. $h_x = 2xy\cos(zt^{-1})$, $h_y = x^2\cos(zt^{-1})$, $h_z = -x^2yt^{-1}\sin(zt^{-1})$, $h_t = x^2yzt^{-2}\sin(zt^{-1})$
35. $\frac{\partial u}{\partial x_i} = \frac{x_i}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$ for all i .
37. 1
39. f is undefined at $(1, 2, 2)$, so the requested quantity does not exist!
41. $\frac{\partial z}{\partial x} = -\frac{x}{3z}$, $\frac{\partial z}{\partial y} = -\frac{2y}{3z}$
43. $\frac{\partial z}{\partial x} = -\frac{yz}{xy-e^z}$, $\frac{\partial z}{\partial y} = -\frac{xz}{xy-e^z}$
- 51a. $\frac{\partial z}{\partial x} = f'(x)$, $\frac{\partial z}{\partial y} = g'(y)$
- 51b. $\frac{\partial z}{\partial x} = f'(x+y) = \frac{\partial z}{\partial y}$
47. $f_{xx} = 6xy(2x-1)$, $f_{xy} = x^2(4x-3) = f_{yx}$, $f_{yy} = 0$
57. $f_{xxx} = 6y(4xy-1)$, $f_{xyx} = 6x(4xy-1)$
59. $f_{xyz} = 2z(1+3xyz^2+x^2y^2z^4)\exp(xyz^2)$
61. $\frac{\partial^3 W}{\partial u^2 \partial v} = \frac{3}{2}v(u+v^2)^{-\frac{5}{2}}$